CSC1310 Foundations of Computer Science

Approximation of π

π is a mathematical constant that is the ratio of any circle's circumference to its diameter. π is approximately equal to 3.14 in the usual decimal notation. Many formulas in mathematics, science, and engineering involve π, which makes it one of the most important mathematical constants. $\pi(3.14159265\ldots)$ is an irrational number, which means that its value cannot be expressed exactly as a fraction having integers in both the numerator and denominator. Consequently, its decimal representation never ends or repeats. (from wikipedia.com)

The following describes a method of approximating the value of π by throwing darts. Suppose that a dart player throws a number of darts at a circular target inscribed into a square and all the darts land inside the square. Figure 1 shows an imaginary distribution of darts on the target. Assume that the chances to hit anywhere inside the square are equal. If a dart lands inside the circle, it is counted as a hit, otherwise a miss. It is observed that the chance (probability) to hit inside the circle is proportional to the ratio of the area of the circle to the area of the square.

A mathematical formula is used to express the relationship:

$$\frac{H}{T} = \frac{\pi R^2}{(2R)^2} = \frac{\pi}{4}$$

where H is the number of hits and T is the total number of darts thrown. Then π can be expressed as:

$$\pi = 4\frac{H}{T}$$

The steps to approximate the value of π:

1. Generate a pair of x and y coordinates of darts that marks the location of a dart hit on the target using a random number generator function called $rand()$. It generate a decimal number between 0 and 1 (exclusive).

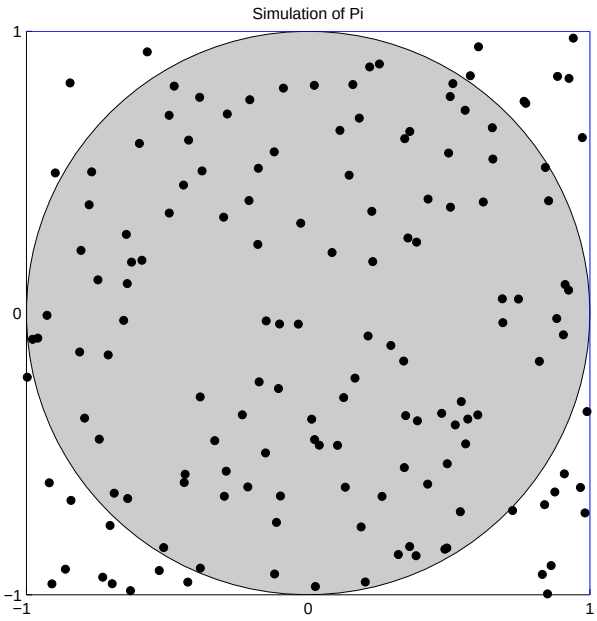

Figure 1: Distribution of Darts on a Target

2. Calculate the distance between the darts and the origin of the coordinate system with the formula below:

$$d = \sqrt{x^2 + y^2}$$

3. Assume that the radius R of the circle target is 1. If d is less than 1, count as a hit, otherwise a miss.

4. Count the number of hits and the total number of darts thrown.

5. Calculate π using the preceding formula $\pi = 4\frac{H}{T}$.